

# Non-deterministic Hypersubstitutions

Jörg Koppitz  
*University of Potsdam*  
*Germany*

A mapping  $\sigma$  which assigns to every  $n$ -ary operation symbol  $f$  an  $n$ -ary term of a given type  $\tau$  is said to be a hypersubstitution of type  $\tau$ . The concept of a hypersubstitution was introduced by Denecke, Lau, Pöschel and Schweigert in the 90th. Every hypersubstitution  $\sigma$  of type  $\tau$  induces a mapping  $\hat{\sigma}$  on the set of all terms of type  $\tau$ . Its application to a term  $t$  is that term which we get from  $t$  by replacing of each operation symbols  $f_i$  in  $t$  by the term  $\sigma(f_i)$ . This process will be more clear if we regard a term as a tree. Now we want to consider the case that one has more than one possibility for the replacement of any operation symbol in a given term  $t$  by only one application of a mapping to  $t$ . This can be realized by the concept of a non-deterministic hypersubstitution. Here the image of any operation symbol  $f_i$  is an element of a given set of terms instead of that for  $f_i$  determined term  $\sigma(f_i)$ . That means, the application of a non-deterministic hypersubstitution to a term gives a set of terms. We will introduce the theory of a non-deterministic hypersubstitution as well as of a non-deterministic solid (*nd-solid*) variety. Further, we give a relationship to the clone theory.